Numerical evaluation of field profile in an undulator with bulk HTS

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Abstract — Application of bulk high-Tc superconductors (HTS) to a permanent magnet undulator is numerically evaluated by macroscopic numerical simulation using the critical state model. Shielding currents are induced by field-cooled magnetization with increasing of the gap length between magnets of the undulator. Field profiles in the undulator are compared for cases with and without the bulk HTS. Numerical results agree well with the experimental results.

I. INTRODUCTION

Permanent magnet (PM) undulators are important devices in synchrotron radiation (SR) facilities. Fig. 1 shows a schematic model of the PM undulator. Magnetic circuits are formed by magnet modules with PM and magnetic adjustments. Periodical alternate magnetic fields are obtained in a gap between the PM modules. Directions of magnetization of the magnets are shown in Fig.1 by arrows. When high speed electrons are injected to the PM undulator, the electron wiggles by Lorentz force like a broken arrow, and radiate a part of its energy as SR. Radius of the synchrotron motion depends on magnitude of magnetic field. Since magnetization of PM is order of 1.0 T, application of a bulk high-Tc superconductor (HTS) to the undulator was reported in the SPring-8, the largest SR facility in Japan, to obtain a short period of SR by high magnetic field using bulk HTS [1]-[2]. Shielding currents are induced in the HTS by field-cooled magnetization, with increasing of the gap length between the magnets of the undulator. There is no external power to magnetize the bulk HTS.

In the present study, the field profiles in the undulator with the bulk HTS are numerically evaluated in the analysis of the field-cooled magnetization. The permanent magnets are expressed using surface currents. Shielding currents in the bulk HTS are analyzed by macroscopic numerical simulation using the critical state model with thin plate-multi layers approximation. Numerical results agree well with the reported experimental results.

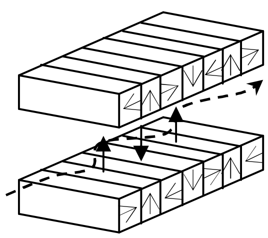


Fig. 1. Simple model of an undulator

II. NUMERICAL FORMULATION

Macroscopic electromagnetic phenomena in HTS are described by Maxwell equations [3]-[5] :

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}, \quad \nabla \cdot \boldsymbol{B} = 0, \tag{1}$$

where μ_0, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the magnetic permeability in air, and the electric and magnetic fields, respectively. The magnetic field $\boldsymbol{B}$ is caused by the external current $\boldsymbol{J}_0$ and the shielding current $\boldsymbol{J}_{SC}$:

$$\boldsymbol{J} = \boldsymbol{J}_0 + \boldsymbol{J}_{SC}. \tag{2}$$

The shielding currents are obtained based on the standard critical state model, where constitutive relationships between the shielding current density $\boldsymbol{J}_{SC}$ and the electric field $\boldsymbol{E}$ are obtained from the force balance on a fluxoid :

$$\boldsymbol{J}_{SC} = J_c(|\boldsymbol{B}|)\frac{\boldsymbol{E}}{|\boldsymbol{E}|} \quad (if \quad |\boldsymbol{E}| \neq 0), \frac{\partial \boldsymbol{J}_{SC}}{\partial t} = \boldsymbol{0} \quad (if \quad |\boldsymbol{E}| = 0). \tag{3}$$

When the electric field $\boldsymbol{E}$ is induced in a local region by change of the magnetic field, shielding currents with the critical current density J_c are obtained. If there is no electric field by the shielding effect, the situation of currents is not changed. Though the critical current density J_c has a strong dependence on magnetic field, Bean model is applied to the present analysis since the critical current density is almost constant in the low cryogenic temperature in experiments. Shielding current distributions are evaluated using current vector potential $\boldsymbol{T}$, where $\boldsymbol{J} = \nabla \times \boldsymbol{T}$. Governing equation is obtained under a thin plate approximation [4].

$$\frac{1}{\sigma}\nabla^2 T - \mu_0 \frac{\partial T}{\partial t} - \frac{\mu_0}{4\pi}\int_S \frac{\partial T_n}{\partial t}\nabla' \frac{1}{R} dS' = \frac{\partial B_0}{\partial t}. \tag{4}$$

The shielding current distribution in the superconductor is obtained stably using the following numerical technique with artificial conductivity [4]-[5]. At first, conductivity in all elements is set to very large value, e.g. 10^{15}[1/Ωm], assuming the superconductor is a very good conductor. If current over the critical current density J_c is obtained, the conductivity of the element is corrected as follows:

$$\sigma_{new} = \frac{J_C}{E} = \sigma_{old}\frac{J_C}{J} \quad if \quad J > J_C, \qquad \sigma_{new} = \sigma_{old} \quad if \quad J \le J_C. \tag{5}$$

Numerical model is discritized using finite element method. Matrix equation is resolved in iterative calculations in each time step, until the maximum current is converged to the critical current density. The obtained self-consistent shielding current distribution is almost satisfied with the non-linear constitutive relationships in (3).

III. Numerical Results

A part of the lower PM module is shown in Fig. 2, where size of the PM and the magnetic adjustments are 20x4x20mm and 20x3.5x17mm, respectively. Size of the HTS plate is 32x7x3mm with a hole of 20x4mm. The permanent magnets are expressed using surface currents. Fig. 3 shows magnetic field on a center line in y-direction in Fig.2. The gap length between lower and upper PM modules is 5.0mm. Both experimental and numerical results agree well, and the maximum magnetic field without HTS plate is about 1.0 T.

The HTS plate is set to the top of the permanent magnet and field-cooled. The gap length between lower and upper PM modules is increased from zero to 20.0 mm as shown in Fig.4. Shielding currents are induced change of the magnetic field. A model of the HTS plate with a hall is divided 30x30x2x7 triangle elements. The hole is treated as low conductivity region. Fig. 5 shows comparison of experimental and numerical magnetic fields on the center line with 5.0mm gap length, where Figs. (a) and (b) are the cases with J_c=1.0X10^8 A/m^2 and J_c=5.0X10^8 A/m^2, respectively. Numerical result of the case with J_c=5.0X10^8 agrees well with the experimental result. Fig. 6 shows comparison of experimental and numerical magnetic fields for different gap lengths, where Figs. (a) and (b) are experimental and numerical results, and dash and solid lines show the cases with and without the HTS, respectively. Decreasing tendency of the field against the gap length is almost the same for both cases.

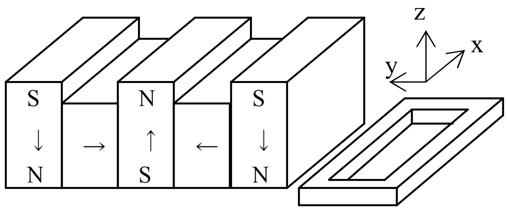


Fig. 2. Permanent magnet module and a HTS plate with a hole

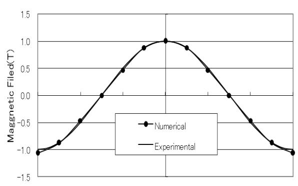


Fig. 3. Magnetic field profile without HTS for 5.0mm gap length

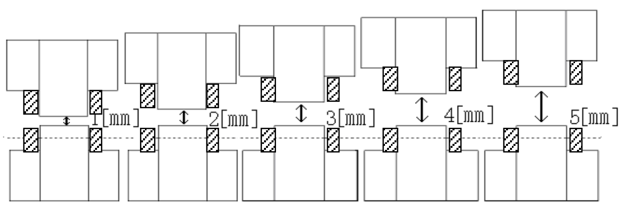
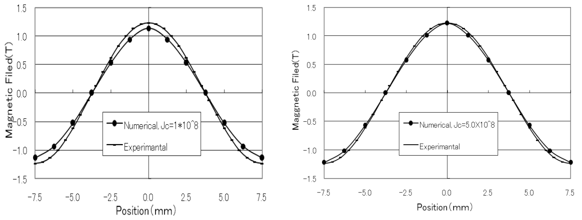
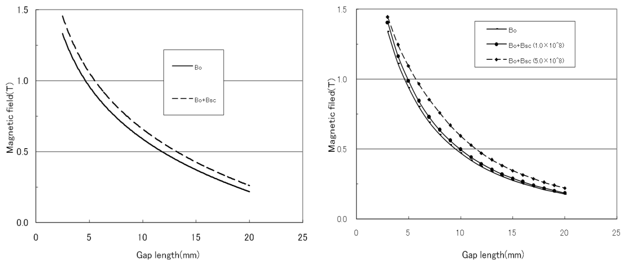


Fig. 4. Field-cooled magnetization by increasing of the gap length



(a) J_c=1.0X10^8 A/m^2 (b) J_c=5.0X10^8 A/m^2

Fig. 5. Magnetic field profile with HTS for 5.0mm gap length



(a) Experimental results (b) Numerical results

Fig. 6. Magnetic fields for different gap lengths: dash and solid lines show the cases with and without the HTS plate.

IV. Coclusion

Field profiles in PM undulator with and without bulk HTS are numerically evaluated in the analysis of field-cooled magnetization. Numerical results agree well with the reported experimental results. Shielding current distributions and size effect of the HTS will be discussed in a full paper.

V. References

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